



$$\beta = 2\alpha \quad (1)$$

$$\gamma > 90^\circ \quad (2)$$

$$a < b < c$$

$$\frac{b}{a} = \frac{\sin \beta}{\sin \alpha} = \frac{2 \sin \alpha \cos \alpha}{\sin \alpha} = 2 \cos \alpha$$

$$\frac{c}{a} = \frac{\sin \gamma}{\sin \alpha} = \frac{\sin(180 - \alpha - \beta)}{\sin \alpha} = \frac{\sin 3\alpha}{\sin \alpha} = 3 \cos^2 \alpha - \sin^2 \alpha = 4 \cos^2 \alpha - 1$$

let $r \triangleq 2 \cos \alpha$

$$\frac{b}{a} = r \quad \frac{c}{a} = r^2 - 1$$

from (1) and (2) we have $\alpha < 30^\circ$. Therefore,

$$0 < \alpha < 30^\circ \Rightarrow \frac{\sqrt{3}}{2} < \cos \alpha < 1 \Rightarrow \sqrt{3} < r < 2$$

$$\Rightarrow 1.73 < r < 2 \quad (3)$$

a, b, c should be integers as small as possible

So, we look for r values that can be represented by a fraction with the smallest possible denominator

$$r \triangleq \frac{m}{d}$$

For $d = 1, 2, 3$ there is no r fulfilling the (3)

For $d = 4, m = 7$ the (3) is fulfilled

$$\frac{b}{a} = \frac{7}{4} \quad \frac{c}{a} = \frac{33}{16} \quad (4)$$

The smallest set of integers (a, b, c) fulfilling (4) is

$$a = 16 \quad b = 28 \quad c = 33 \quad \text{for a perimeter of } \underline{\underline{77 \text{ meters}}}$$

For $d \geq 5$ no solution with perimeter < 77 can be found, because we have $a \geq d^2$ and for $a = 25$ we would already have a minimum perimeter of $(25 + 26 + 27) = 78$ meters.