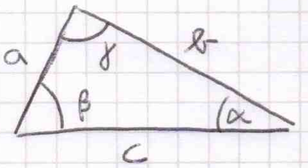
ANDREA ANDENNA 9/5/2010



$\beta = 2\alpha \quad (1)$

$\gamma > 90° \quad (2)$

$a < b < c$

$$\frac{b}{a} = \frac{\sin\beta}{\sin\alpha} = \frac{2\sin\alpha\cos\alpha}{\sin\alpha} = 2\cos\alpha$$

$$\frac{c}{a} = \frac{\sin\gamma}{\sin\alpha} = \frac{\sin(180-\alpha-\beta)}{\sin\alpha} = \frac{\sin 3\alpha}{\sin\alpha} = 3\cos^2\alpha - \sin^2\alpha = 4\cos^2\alpha - 1$$

let $r \triangleq 2\cos\alpha$

$$\frac{b}{a} = r \qquad \frac{c}{a} = r^2 - 1$$

from (1) and (2) we have $\alpha < 30°$. Therefore,

$$0 < \alpha < 30° \Rightarrow \frac{\sqrt{3}}{2} < \cos\alpha < 1 \Rightarrow \sqrt{3} < r < 2$$

$$\Rightarrow 1.73 < r < 2 \qquad (3)$$

a, b, c should be integer as small as possible

So, we look for r values that can be represented by a fraction with the smallest possible denominator

$$r \triangleq \frac{n}{d}$$

For $d = 1, 2, 3$ there is no r fulfilling the (3)

For $d = 4$, $n = 7$ the (3) is fulfilled

$$\frac{b}{a} = \frac{7}{4} \qquad \frac{c}{a} = \frac{33}{16} \qquad (4)$$

The smallest set of integers (a, b, c) fulfilling (4) is

$a = 16 \quad b = 28 \quad c = 33$ for a perimeter of <u>77 meters</u>

For $d \geq 5$ no solution with perimeter < 77 can be found, because we have $a \geq d^2$ and for $a = 25$ we would already have a minimum perimeter of $(25+26+27) = 78$ meters.