

FSJM – SEMI-FINAL - March 12th 2016

Information and results at <http://fsjm.ch/>

START for ALL PARTICIPANTS

1 – 2016 the year that makes a difference (coefficient 1)

Using the four digits 2, 0, 1 and 6, we can make two two-digit numbers and calculate their difference.

For example, $26 - 10 = 16$ or $20 - 16 = 4$.

4 is the smallest possible difference.

What is the largest possible difference?

Beware: a two-digit number cannot start with 0.

2 – Waiter, the bill please! (coefficient 2)

At the restaurant, Mathew pays for two drinks with a banknote. The waiter gives him two 1 Franc coins and a 10 centime coin in change. But he has made a mistake, he should have given Mathew two 10 centime coins and a single 1 Franc coin.

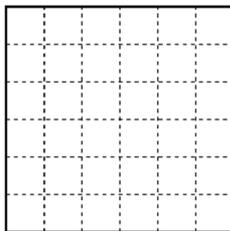
How much extra change did he give ?

3 – A square in nine (coefficient 3)

It's easy to divide a square into nine identical smaller squares.

Divide this square into nine squares that are not all the same size.

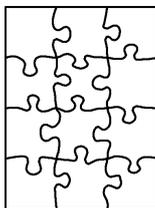
Cuts must only be along the gridlines.



4 – Sacha's puzzle (coefficient 4)

Sacha has made the 12-piece rectangular puzzle in the picture.

Now he is going to make a second rectangular puzzle of the same sort of a picture of a cat in its basket, and which has 77 pieces.



How many pieces of the second puzzle will have exactly one straight edge?

5 – Never two without three (coefficient 5)

How many whole numbers between 100 and 1000 which contain the digit 2 also contain the digit 3?

END for CE PARTICIPANTS

6 – In 80 days? (coefficient 6)

In his trip around the world, Phileas Fogg has already travelled 34215 kilometres. This number is made of five consecutive digits (i.e. digits that follow one another). At this point he has 5785 km left to travel to finish his trip around the world.

When Phileas has travelled the greatest number of kilometres that can be made from five consecutive digits, how many kilometres does he have left to travel to finish his trip around the world?

7 – Mathew's collection (coefficient 7)

Mathew wants to put numbers on the toy cars in his collection (he has more than 100 of them). To do this, he buys stickers bearing the digits 0, 1, 2, 3, 4, 5, 6, 7 or 8; the digit 6 can represent a 9 by being used upside down. He has 20 stickers of each digit, so 180 stickers altogether.

If Mathew numbers his cars from 1 upwards, what will be the first number that he cannot make?

8 – At the masked ball, dance with them all? (coefficient 8)

There were 31 people at the masked ball.

Emma danced with 8 boys, Jade danced with 9 boys, Chloe danced with 10 boys, etc.... up to Manon, the last girl, who danced with all the boys that were there.

How many boys were there ?

END for CM PARTICIPANTS

Problems 9 to 18 : beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give any two correct solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be just one solution).

9 – Generation game (coefficient 9)

For every day of the year we associate a number, formed by the number of the day in the month followed by the number of the month (a number never starts with the digit 0). Baptiste tells us his birthday number is 131. His birthday is the 13th of January.

His great grandfather tells us his own birthday number and from it we cannot tell his birthday.

What is Baptiste's great grandfather's birthday number?

10 – Du café noir (coefficient 10)

A cryptarithm is a substitution code where different letters represent different digits, two digits can never be represented by the same letter and numbers cannot start with a 0.

$$\text{DU} + \text{CAFE} = \text{NOIR}$$

This cryptarithm has more than one solution. **But for the solution where CAFE is the smallest possible number, what number is NOIR ?**

11 – Cedric's age (coefficient 11)

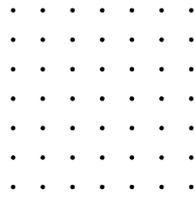
We are in the year 2016, and Cedric's age is a factor of 2016. If Cedric adds up all the multiples of his age that are less than 365, he arrives at the year he was born.

In which year was he born?

END for C1 PARTICIPANTS

12 – Forty-nine points (coefficient 12)

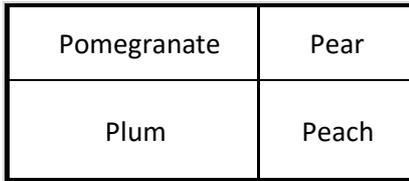
49 points are marked on a sheet of paper. Adjacent points horizontally or vertically are separated by exactly 1 centimetre.



How many straight lines of length 5 centimetres can be drawn between points in the design?

13 – P-cake (coefficient 13)

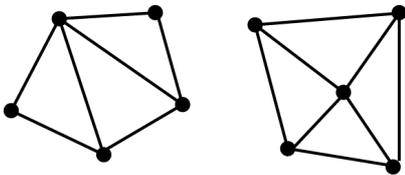
Philippe has baked a rectangular cake, made of four rectangular portions. In every portion there is a different fruit flavour, as shown in the following drawing.



An ant has travelled around the perimeter of every rectangle formed by two flavours which touch along a side. The four perimeters are 82 cm, 74 cm, 92 cm and 94 cm.

What is the perimeter of Philippe’s cake?

14 – I triangulate, you triangulate, ... (coefficient 14)



In his exercise book, Mathew has drawn 5 points, then he has connected some points to make a grid only of triangles whose vertices are his original points. Thus he makes three triangles (the figure on the left). Matilda does the same task and makes four triangles from her points (the figure on the right).

If you draw 2016 points on a sheet of paper, what is the maximum number of triangles you can obtain, without any triangles overlapping?

END for C2 PARTICIPANTS

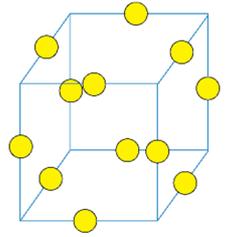
15 – Who bids higher? (coefficient 15)

Matt the Kid counts the number of factors of the number 2016. He finds that there are many of them.

Which year of the third millennium has even more factors?

16 – Game cube (coefficient 16)

In a videogame, every edge of a cube carries a ball containing a number of gold pieces. All 12 balls contain a different whole number of gold pieces between 1 and 12. The contents of every ball is hidden.



A game consists of choosing a vertex and then travelling an unbroken path along three different edges, collecting the gold pieces along the way. The numbers of gold pieces collected from every edge travelled must be in increasing order to win the game. If the order is not increasing, the game is lost.

What is the minimum number of winning paths on the cube?

Two paths are considered distinct if they differ by at least one edge.

END for L1, GP PARTICIPANTS

17 – The houses of Maths-beach (coefficient 17)

Along the coast of Maths-land, the straight beach-front road contains a line of houses, all on the same side of the road. The houses are painted either blue or yellow and there is at least one house of each colour. Curiously enough, every pair of houses separated by ten other houses is painted the same colour, as is every pair separated by fifteen houses.

What is the maximum number of houses on this road?

18 – Mathew’s questions (coefficient 18)

Matilda has written the series of squares of all whole numbers larger than 1 and smaller than or equal to 1000: 4 ; 9 ; 16 ; 25 ; ; 998 001; 1 000 000.

For every square a^2 Matilda has written, Mathew checks if there is a whole number n bigger than 1 and smaller than a for which a^2 is a multiple of $n(2an - n^2) + 1$.

What is the number written by Matilda for which Mathew can find a whole number n for which the condition is satisfied?

END for L2, HC PARTICIPANTS